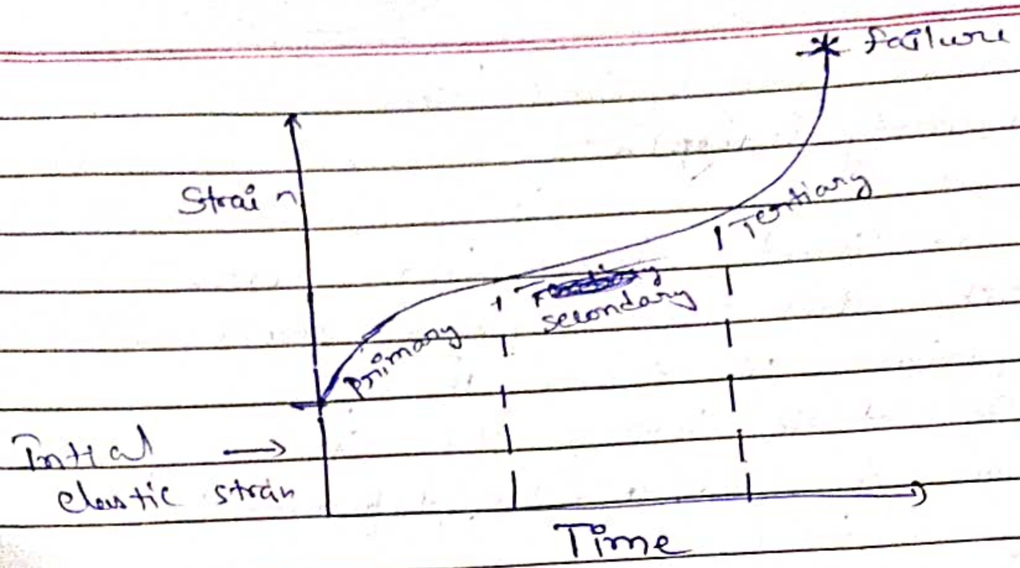

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Stress and strain:-

As per concern of mining there are two types of stresses:-

(a) In-situ stress (Pre-mining stress):-

$$\boxed{\sigma_v = \gamma L}$$

γ = average wt. density of ovelying strata (kN/m^3)
Z = depth of cover

(b) Induced stress (Equilibrium stress):-

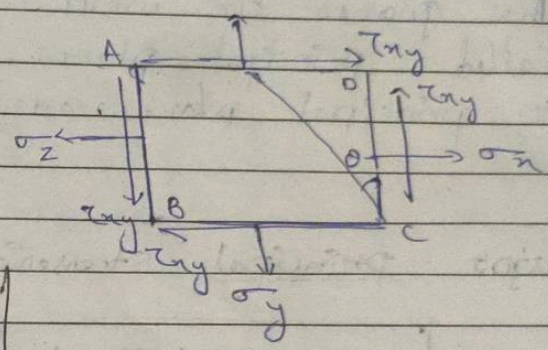
Stress resulting from redistribution of stresses after excavation made in rock mass.

Normal stress(σ) and shear stress (τ)

Normal stresses act perpendicular to the surface of body whereas shear stresses act parallel (tangential) to the surface of a body (parallel to plane).

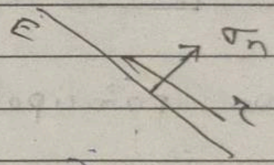
Two dimension state of stresses acting on a body:-

Normal stress (σ_n)
and shear stress (τ).



On plane CE making angle θ with CD

$$\boxed{\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta}$$



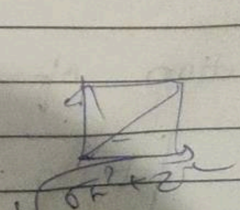
$$\boxed{\tau = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$$

$-\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

$$\boxed{\text{Resultant stress} = \sqrt{\sigma_n^2+\tau^2}}$$

Angle with which resultant stress makes with x-axis or σ-axis

$$\boxed{\phi = \tan^{-1}\left[\frac{\tau}{\sigma_n}\right]}$$



Normal stresses act perpendicular to the surface of body whereas shear stresses act parallel (tangential) to the surface of a body (parallel to plane).

Two dimension state of stresses acting on a body:-

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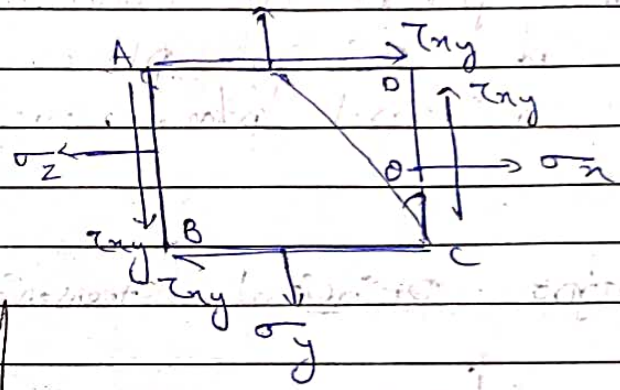

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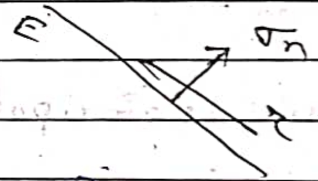

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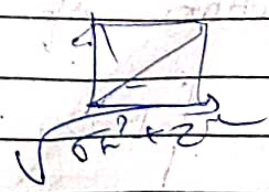

$\frac{\sigma_2}{\sigma_1}$ 90°

Sign conventions:-

Tensile stress is taken positive and compressive stress is taken negative

Clockwise shear stress is taken positive whereas anticlockwise taken as negative

The planes on which shear stress are zero are called principal planes and stresses normal to principal plane are principal stresses.

$\sigma_1 \sigma_2$

Major principal stresses:-

$$\boxed{\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

Minor principal stress:-

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\boxed{\sigma_1 + \sigma_3 = \sigma_x + \sigma_y}$$

Direction of principal planes:-

$$\boxed{\tan 2\theta = \left|\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right|}$$

we get two values of θ differ each other by 90° i.e. θ_1, θ_2 $\quad \theta_1 = \theta_2 \pm 90°$

σ_1 , $\frac{\sigma_1 - \sigma_2}{2}$

Directions of maximum shear stress with plane σ_x (on x-axis) $\leq 45 + \theta$,

Maximum shear stress = $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_x , σ_y

Biaxial stress condition:-

shear stress is absent ($\tau_{xy} = 0$)

$\sigma_1 = \sigma_x$

$\sigma_2 = \sigma_y$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

when $\tau_{xy} = 0$

$$\boxed{\begin{aligned} \sigma_1 &= \sigma_x \\ \sigma_3 &= \sigma_y \end{aligned}}$$

$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$ at $\theta = 45°$

* Pure shear:-

$$\boxed{\sigma_n = \tau_{xy} \sin 2\theta}$$

$$\tau = -\tau_{xy} \cos 2\theta$$

$$\boxed{\sigma_1 = \sigma_3 = \tau_{xy}}$$

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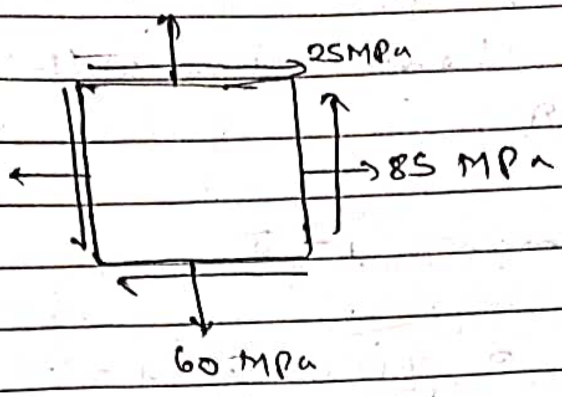
Q.

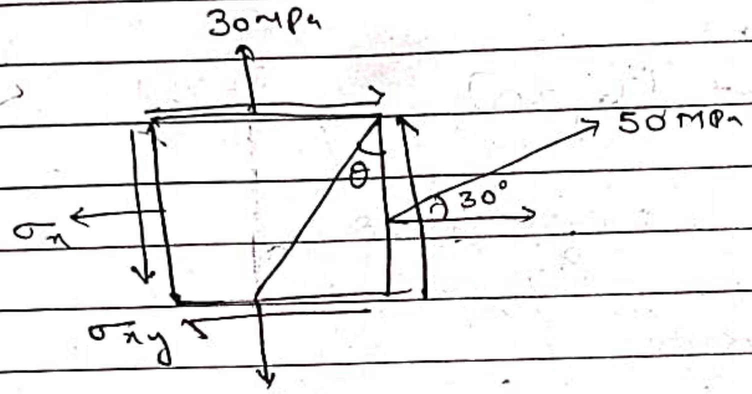


$\sigma_1 = 122 MPa$

$\sigma_3 = -12 MPa$

$\theta_1 = 31°7', 121°7'$

Q)



$\sigma_x = 50 \cos 30° = 43.3 MPa$

$\tau_{xy} = 50 \sin 30 = 25 MPa$ $\quad$ σ_1

$\sigma_y = 30 MPa$

Direction $= \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{43.3 + 30}{2} + \sqrt{\left(\frac{43.3 - 30}{2}\right)^2 + 25^2}$$

$$= 63.85 \text{ MPa}$$

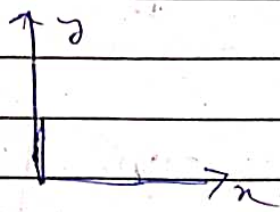
$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau_{xy}^2}$$

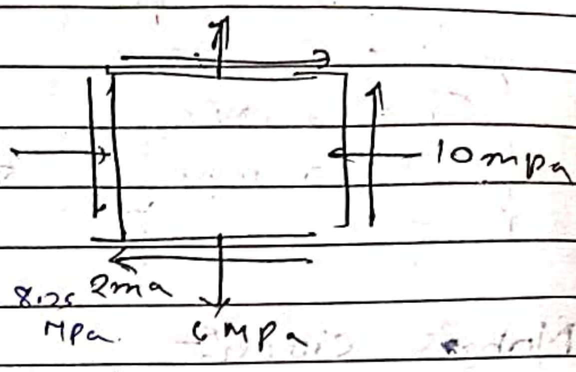
Q. Find the magnitude and direction of max^m shear stress for the following stress condition

$\sigma_x = 10 MPa$

$\sigma_y = 6 MPa$

$\tau_{xy} = 2 MPa$



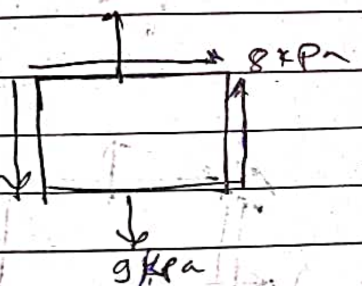


$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 8.25 \text{ MPa}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \theta_1 = -7.018^\circ$$

Direction of $\tau_{max} = 45^\circ + \theta_1 = 37.98^\circ$

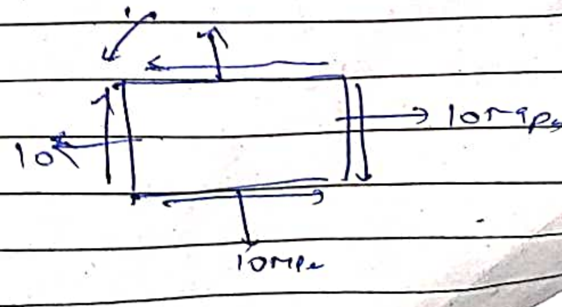
Q)



Direction of $\tau_{max} = \tan^{-1}\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) = 37^\circ 58' 55''$

$\sigma_1, \sigma_3, \theta, \tau_{max}$ - ?

Q) For the state of stress (MPa) as shown in the figure, the major principal stress is 15 mpa Find shear stress.



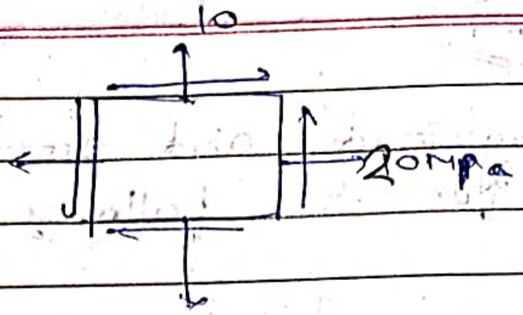
Sol^n:- $\sigma_x = 10 MPa$, $\sigma_y = 10 MPa$

$\sigma_1 = 15 MPa$

$\tau_{xy} = ?$

Q. Find

σ_1 & σ_3

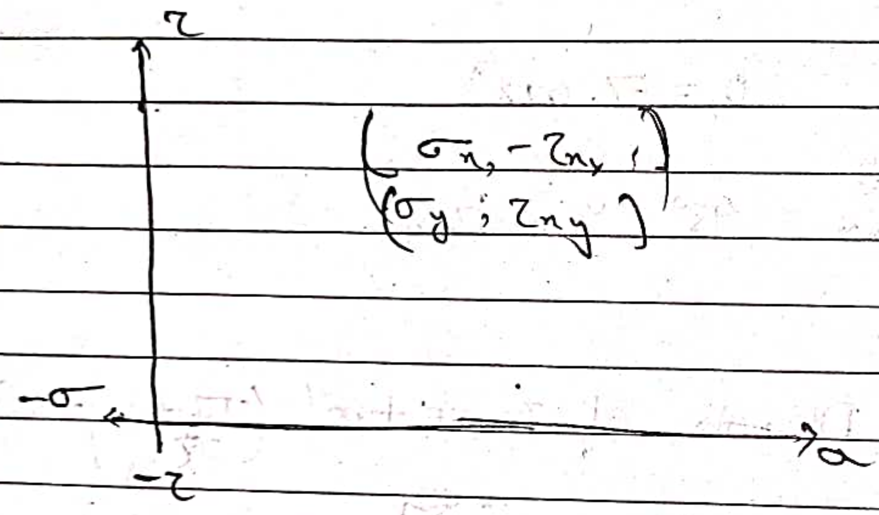


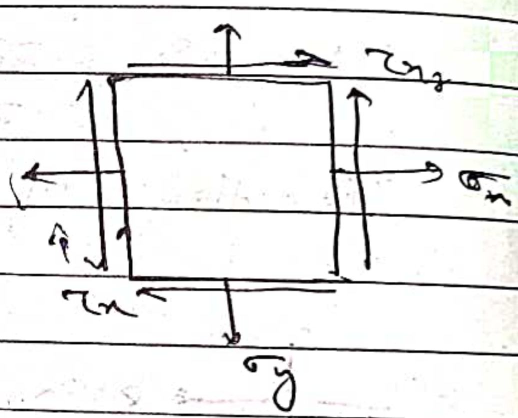
Imp

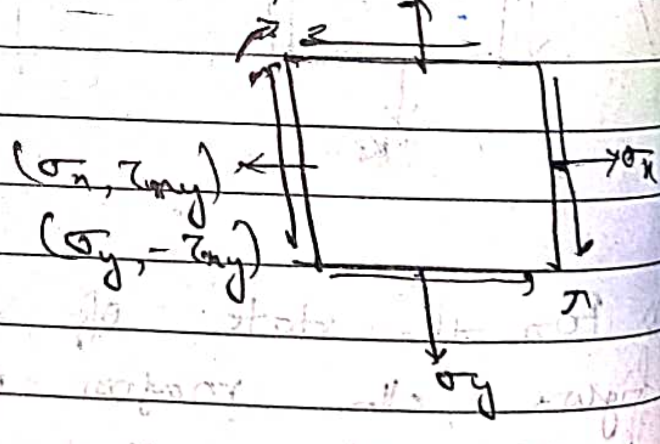
$\sigma_1 = 20$ MPa
$\sigma_3 = 10$ mpa $\qquad \tau_{max} = 5$ MPa

→ Direct axes

Mohr's circle:-







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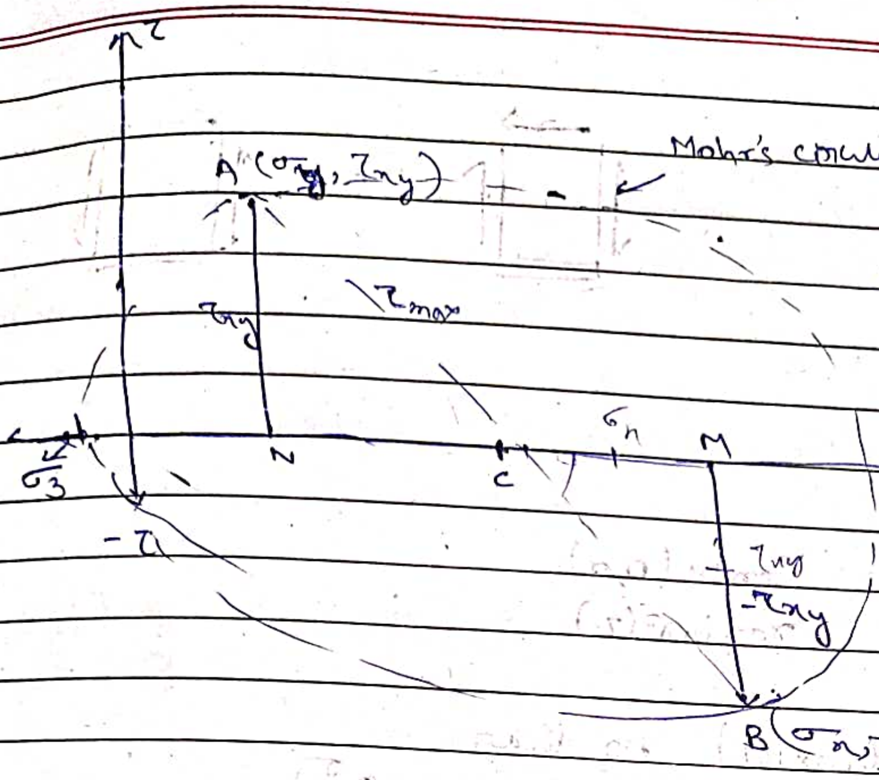


C - centre of mohr's circle

CA = CB = (radius of circle)

$$\boxed{\text{Centre} = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)}$$

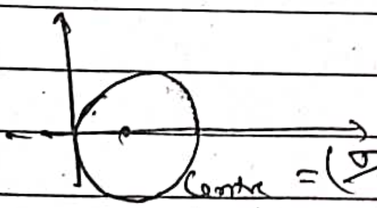
$$\text{Radius } (\tau_{max}) = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \left(\frac{\sigma_1 - \sigma_3}{2}\right)$$

Circle intersects σ axis at two points known as σ_1 & σ_3

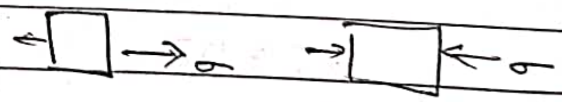
CA = CB = (radius of circle)

Mohr's circle in different conditions of stress :-

① uniaxial tension or compression



Centre = $(\sigma/2, 0)$
radius = $\sigma/2$

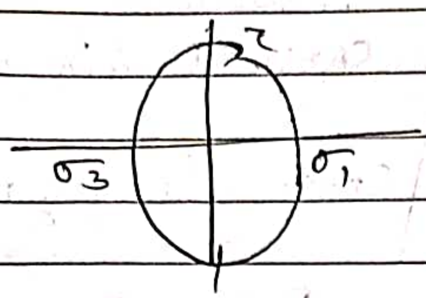


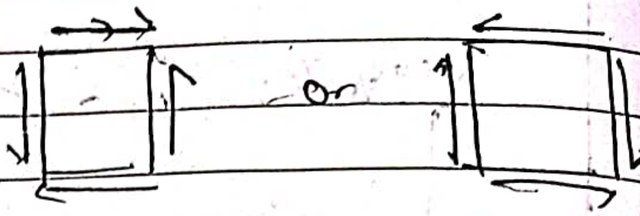
$$\boxed{\sigma_1 = \sigma \qquad \sigma_3 = 0}$$

$$\boxed{\tau_{max} = \frac{\sigma}{2}}$$

$\sigma_1 = -\sigma$
$\sigma_3 = 0$
$\tau_{max} = \sigma$

② Pure shear



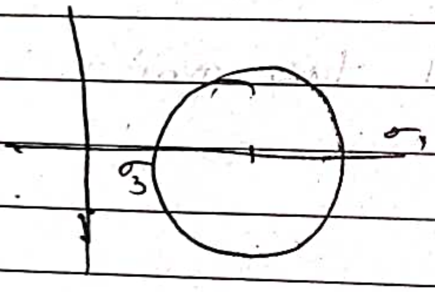


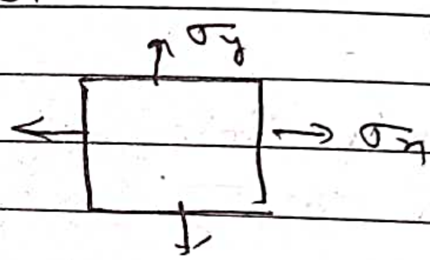
$\sigma_1 = \tau$

$\sigma_3 = -\tau$ Centre $(0,0)$

$\tau_{max} = \tau$ radius $= (\tau)$

③ Normal stress (biaxial) no shear





Centre $= \left(\frac{\sigma_x + \sigma_y}{2}, 0 \right)$

Radius $= \frac{\sigma_x - \sigma_y}{2} = \tau_{max}$

$\sigma_1 = \sigma_x$

$\sigma_3 = \sigma_y$

If $\sigma_x = \sigma_y$ Circle becomes a dot (point) at $(\sigma_x$
(Hydrostatic condition)

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Deformation in rock mass:-

The amount of deformation is called strain. The type and amount of strain that a rock experiences depends on:-

1> Type of stresses applied - Normal, shear, all round pressure etc.

2> Depth and temperature - vertical stress and Thermal stress

Normal strain (ϵ) $= \frac{\sigma}{E}$

Shear strain (γ) $= \frac{\tau}{G}$

Volumetric strain (ϵ_v) $= \frac{\Delta V}{V} = \frac{P}{K}$ → volumetric press. ; K → Bulk modulus

P = pressure

$\frac{0.05}{18.42 \times 10^{4}}$

$\frac{0.05 \times 10^{4}}{18.42}$

$\frac{0.05 \times 10^{4} \times 2}{36.84}$

$= \frac{0.1 \times 10^{4}}{36.84}$

Generally rocks are heterogenous, anisotropic and brittle. But we consider rock as homogenous isotropic and elastic

Partially rocks show elastic & brittle behaviour near the earth's surface and show more plastic and ductile behaviour deeper down the earth because of Increasing temperature and pressure.

Failure in rocks:- Failure in rock may include both fracture and flow. There are three types of failures.

(i) Rupture :- It occurs when ductile material fails in tension

(ii) Brittle fracture - It occurs in rock due to tension

(iii) shear ~~friction~~ fracture:- It occurs in rocks when they are subjected to compression. Rocks are weaker in tension and stronger in compression. So, compressive strength of rock is very high compared to tensile strength.

Cohesion:- It is the force of attraction among same type of particles in a material

Internal friction:- It is property of a material due to which the particles in a layer opposes the movement of particles in upper layer.

The existence of universe is due to these two properties of material

Failure Criteria:-

Rocks fail when stresses exceeds their stress bearing capacity (strength). Failure occurs by the development of fracture. Failure can occur along a single fracture or network of fractures.

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There are following criteria for failure of rock mass-

① Maximum shear stress Criterion (Tresca):-

According to this criterion, a material fails when the maximum shear stress applied on rock mass becomes equal to maximum shear strength of rock mass becomes equal to the maximum shear strength of rock mass. The failure planes are inclined at an angle of 45° with direction of principle stress.

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}}$$

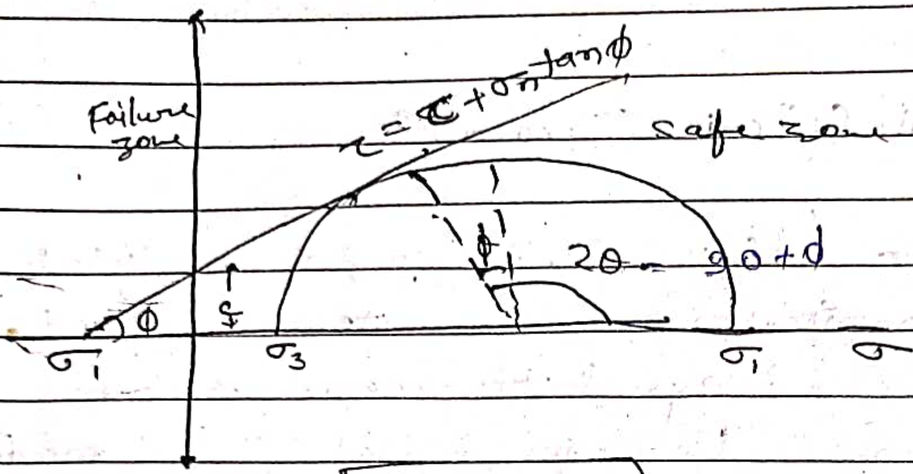
② Mohr- Columb criterion:-

It is linear shear failure criterion and consider cohesion and angle of internal friction. According to this criterion a failure occurs along a shear failure plane and shear strength of rock are made up of two parts i.e cohesion and frictional component of normal stress

$$\tau = C + \sigma_n \tan\phi$$

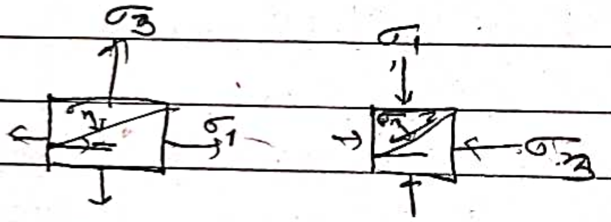
C = cohesion, ϕ = angle of internal friction.

Failure occurs at $45° + \frac{\phi}{2}$



$$\boxed{Y = mx + c}$$

$$\tau = c + \sigma_n \tan\phi$$
$$Y = c + x \; m$$



σ_c / σ_t depends on case

$$\boxed{\sigma_1 = \frac{2c\cos\phi}{1-\sin\phi} + \sigma_3 \frac{1+\sin\phi}{1-\sin\phi}}$$

Uniaxial compressive strength

$$\sigma_c = \frac{2c\cos\phi}{1+\sin\phi}$$

Uniaxial tensile strength

$$\sigma_t = \frac{2c\cos\phi}{1-\sin\theta}$$

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$$\frac{\sigma_c}{\sigma_t} = \frac{1-\sin\phi}{1+\cos\theta}$$

$2\theta = 90 + \phi$

$\boxed{\theta = 45 + \frac{\phi}{2}}$ → failure plane

3) Hock-brown Failure criterion:-

This criterion is also called empirical failure criterion as it is based on lab experiments.

According to this criterion, the peak triaxial compressive strength of a wide range of rock material is related to confining stress and given as

$$\sigma_1 = \sigma_3 + \sigma_c \left(m\frac{\sigma_3}{\sigma_c} + s\right)^a$$

where σ_1, σ_3 are peak triaxial stress, σ_c - uniaxial compressive stress, m, s and a are rock parameters

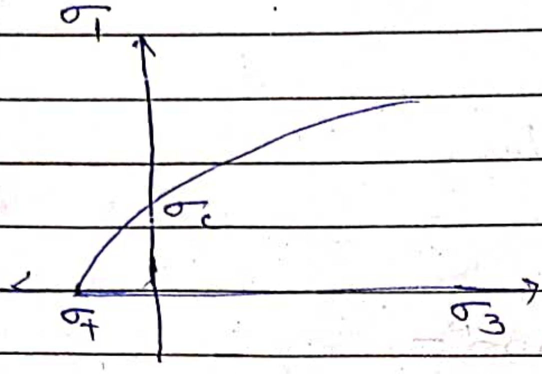
$a = 0.5$ & $s = 1$, so the equation becomes

$$\boxed{\sigma_1 = \sigma_3 + \sigma_c\left(m\frac{\sigma_3}{\sigma_c} + 1\right)^{0.5}}$$

or $\boxed{\sigma_1 - \sigma_3 = (m\sigma_3\sigma_c + \sigma_c^2)^{0.5}}$

value of m ranges from 7 to 25. 7 for rocks

Containing more number of joints, 25 for igneous & metamorphic rocks.



Griffith's criterion:-

Originaly conducted on glass, then extended to rock m. It is based on mechanics of brittle fracture using elastic strain energy concepts. According to the criterion tensile failure is brittle material initiates at crack tips. Under compression the crack propagates from the point of maximum tensile stress concentration.

$(\sigma_1 - \sigma_3)^2 = 8\sigma_T(\sigma_1 + \sigma_3)$ if $\sigma_1 + 3\sigma_3 > 0$

if $\sigma_1 + 3\sigma_3 < 0$

when $\sigma_3 = 0$ $\sigma_1 = 8\sigma_T$

i.e uniaxial compressive stress at crack extension is 8 times uniaxial tensile stress

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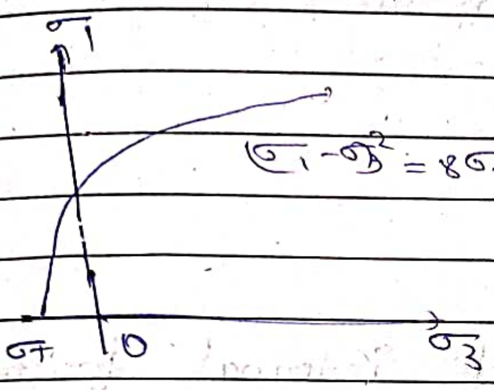
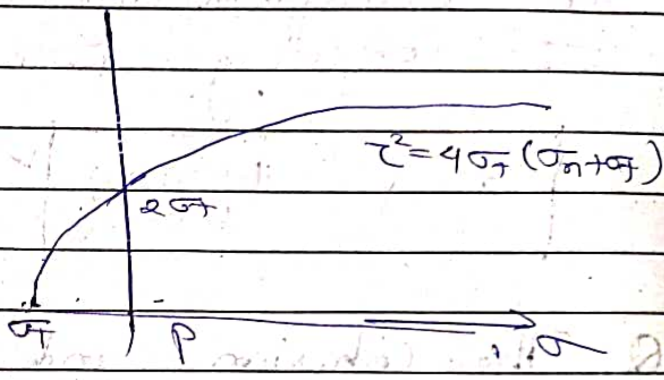
This criterion has also been expressed in terms of shear stress and normal stress as

$$\tau^2 = 4\sigma_T(\sigma_n + \sigma_T)$$





Both curves are in parabolic shape

Triaxial compression test:-

In this test a rock specimen is subjected to uniaxial compressive stress accompanied with biaxial lateral stresses.

According to this test the relationship between major and minor principal stress can be expressed as

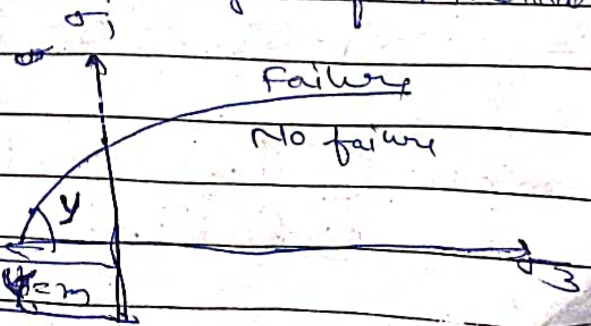
$$\sigma_1 = C_0 + m\sigma_3$$

c - cohesion

ϕ = angle of internal friction

$$C_0 = \frac{2c\cos\phi}{1-\sin\phi}$$

$$m = \frac{1+\sin\phi}{1-\sin\phi} = 45 + \frac{\phi}{2}$$



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Confined Compressive strength (P) :-

$$P = \sigma_c + \sigma_p \cdot \tan \varphi$$

σ_c = Unconfined / uniaxial compressive strength
σ_p = Confining pressure

$$\boxed{\tan \varphi = \frac{1+\sin\phi}{1-\sin\phi}}$$

Q. The cohesion and angle of internal friction of a sand stone rock sample are 6 MPa & 35° respectively. Calculate the unconfined compressive strength and tensile strength of the specimen.

$C = 6\,MPa$
$\phi = 35$

Soln:- $\sigma_c = \frac{2C\cos\phi}{1-\sin\phi}$, $\sigma_t = \frac{2C\cos\phi}{1+\sin\phi}$

$= 23.05\,MPa$, $\sigma_t = 6.25\,MPa$

Q) Rock mass parameter for a shale rock : m, s and a of the Hock - Brown Criterion as given in the eqn are 2.4, 0.01 and 0.50 respectively. The uniaxial compressive strength of intact shale is 50 MPa.

Soln:- $$\sigma_1 - \sigma_3 = \sigma_c \left(m \frac{\sigma_3}{\sigma_c} + s\right)^a$$

Determine ① The principle stress at failure
② Rock mass strength of shale

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Sol^n:- Given:- $m = 2.4$, $S = 0.01$, $a = 0.5$, $\sigma_c = 50$ MPa
$\sigma_3 = 0$ (not given so it is taken as 0)

Now $\sigma_1 = 50(0 + 0.01)^{0.5}$
$= 5$ MPa

$\boxed{\text{Rock mass strength} = \sqrt{m\sigma_c + S^2} \text{ MPa}}$
$= \sqrt{120 + (0.01)^2} = 10.95$ MPa.

Q) Laboratory tests on specimen of a sandstone have produced unconfined compressive and tensile strength 80 MPa. and 10 MPa respectively. Using the hoek-brown and Griffin's criteria estimate the maximum Principal stress at failure for two biaxial test in which $\sigma_3 = 20$ MPa & $\sigma_3 = 40$ MPa, Take $m = 7.88$, $S = 1$ and $a = 0.5$

Sol^n:- ① Hoek - Brown Criterion:-

$$\sigma_1 = \sigma_3 + (m\,\sigma_c\sigma_3 + S\sigma_c^2)^{0.5}$$

$$= \sigma_3 + (7.88\,\sigma_c\sigma_3 + \sigma_c^2)^{0.5}$$

At $\sigma_3 = 20$ MPa, $\sigma_1 = 20 + (7.88 \times 80 \times 20 + 80^2)^{0.5}$ MPa
$= 157.80$ MPa

At $\sigma_3 = 40$ MPa, $\sigma_1 = 40 + (7.88 \times 40 \times 80 + 80^2)^{0.5}$
$= 217.80$ MPa.

S- Comb
(a) P-2,
(c) P-2
48. Pressu

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Griffith's criterion:-

$(\sigma_1 - \sigma_3)^2 = 8\sigma_t(\sigma_1 + \sigma_3)$

So, $(\sigma_1 - 20)^2 = 8 \times 10(\sigma_1 + 20)$

solving, we get $\sigma_1 = 129.3$ MPa

At $\sigma_3 = 40$ MPa

$(\sigma_1 - 40)^2 = 8 \times 10(\sigma_1 + 40)$

Solving, we get $\sigma_1 = 169.40$ MPa

Q. A series of triaxial compression test has been conducted on sandstone sample reveals the following relationship between major and minor principal stresses.

$\sigma_1 = 50 + 3\sigma_3$

Calculate (a) cohesion
(b) Angle of internal friction
(c) Unconfined compressive strength
(d) tensile strength

major principal stress - confined compressive strength
minor " " - confining pressure

Comparing the above eqn with $\sigma_1 = C_0 + m\sigma_3$

we get $C_0 = 50$ and $m = 3$

$C_0 \rightarrow$ compressive stre

$C_0 = \frac{2C\cos\phi}{1-\sin\phi}$ and $m = \frac{1+\sin\phi}{1-\sin\phi}$ From this we get

$C = 14.43$ MPa

$\phi = 30°$

If rock specimen contains joints and undergoes uniaxial compression or tension then total displacement (strain)

= strain due to normal stress + strain due to joint stiffness (normal)

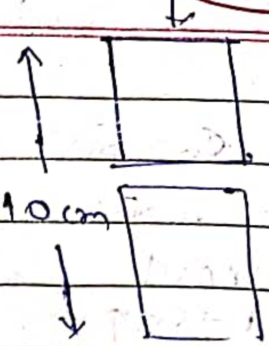
Strain due to normal stress $= \frac{\sigma}{E} \cdot L$

Strain due to joint $= \frac{\sigma}{\sigma_J}$

σ = applied normal stress (MPa)
E = Modulus of elasticity of rock (MPa)
L = Length of specimen (m)
σ_J = joint stiffness or normal stiffness of joint (MPa/m

Q. A rock specimen with a horizontal joint is subjected to 10 MPA of normal stress. The elastic modulus and poisson's ratio of rock are 50 GPa and 0.25 respectively. If the normal stiffness of joint is 50 GPa/m. Calculate the displacement in the sample.

Sol^n:- $\epsilon_{normal} = \frac{\sigma}{E} \cdot L$



$= \frac{10 \times 10^6}{5 \times 10^9} \times 0.1 m$

$= 0.2$ mm

$\epsilon_{joint} = \frac{\sigma}{\sigma_j} = \frac{10 \times 10^6}{50 \times 10^9}$ m

$= 0.2$ mm

$\epsilon_{total} = 0.4$ mm

* Weight Stress:- (vertical stress):-

It is the weight of overlying strata on various mining structures (openings, gallery, pillars, supports etc).

$$\boxed{\sigma_v = \gamma H \text{ (MPa)}}$$

γ - Weight density of overlying strata (kN/m³)
H - Depth of cover

If strata contains different layers of different rocks then,

$$\sigma_v = \gamma_1 H_1 + \gamma_2 H_2 + \dots\dots + \gamma_n H_n$$

γ_1, γ_2 .... - Wt density of individual layers
H_1, H_2 .... - Thickness of each layer

At considerable depth, there is also a horizontal or lateral stress (σ_H)

$$\boxed{\sigma_H = \frac{\mu \sigma_v}{1-\mu}} \qquad \gamma = \text{Poisson's ratio}$$

Q. What will be the value of horizontal stress at point P in the strata as shown below.

$\updownarrow$ 50m $\quad \gamma = 15\,kN/m^2 \quad \gamma = 0.15$

$\updownarrow$ 50m $\quad \gamma = 25\,kN/m^2 \quad \nu = 0.21$

Soln:- $\sigma_v = \gamma_1 H_1 + \gamma_2 H_2 = 19 \times 50 + 25 \times 50$ KPa

$$\sigma_H = \frac{\nu}{1-\nu}\,\sigma_v = \frac{0.21}{1-0.21} \times 2.2 \text{ MPa} = 0.58 \text{ MPa}$$

($\nu = 0.21$ will be taken because P lies on lower strata)

Q> A typical case of gravity loading under complete lateral restraint in a flat strata as shown in the figure. The psycho-mechanical parameters are given in the table. Determine the in-situ stresses (σ_v, σ_H) on the top of the coal seam.

↓ ↓ ↓ ↓ ↓ ↓ ↓ 10 MPa

Sandstone

Shale

Coal

Strata	Thickness	Sp. gr	E (GPa)	G (GPa)
Sandstone	50	2.35	26.40	12.50
Shale	50	2.15	20.50	8.25
Coal	50	2.41	2.41	0.95

Sol^n:- σ_v (on the top of the coal)

$= 10 \text{ MPa} + 2.35 \times 9.81 \times 10^{-3} \times 50 + 2.15 \times 25 \times 9.81$

$= 11.68 \text{ MPa}$

$\nu = \frac{E}{2G} - 1$ $\quad$ $G = \frac{E}{2(1+\nu)}$

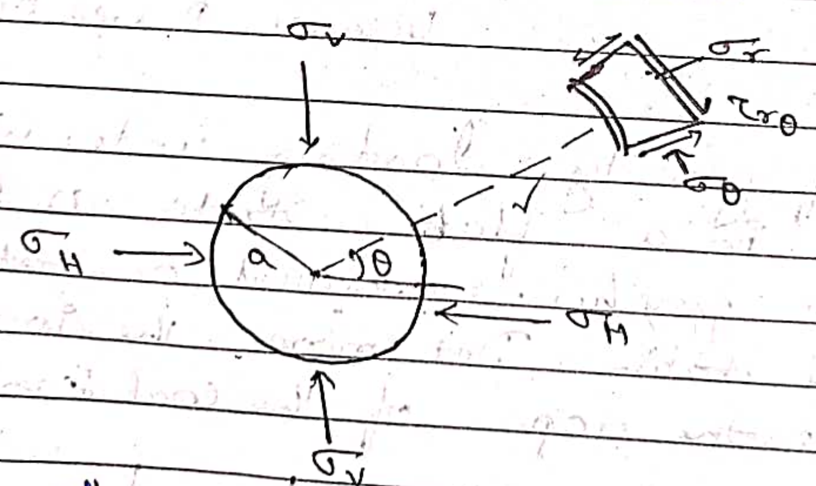
For shale,

$\nu = \frac{20.50}{16.50} - 1 = 0.24$

$\sigma_H = \frac{\nu}{1-\nu} \times \sigma_v = \frac{0.24}{0.76} \times 11.68 \text{ MPa}$

$= 3.69 \text{ MPa}$

* Stress concentration around circular opening made at a considerable depth from surface



This is a circular opening of radius a we have to calculate stresses due to this opening at point r distant apart from the centre of ope

Radial stress $(\sigma_r) = \frac{\sigma_v}{2}\left[(1+k)\left(1-\frac{a^2}{r^2}\right) - (1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$

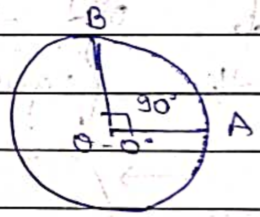
Tangential stress $(\sigma_\theta) = \frac{\sigma_v}{2}\left[(1+k)\left(1+\frac{a^2}{r^2}\right) + (1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$

shear stress $(\tau_{r\theta}) = \frac{\sigma_v}{2}\left[(1-k)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right]$

Cases ① At $r=a$ (At the boundary of the opening)

$$\sigma_r = \tau_{r\theta} = 0$$

$$\sigma_\theta = \sigma_v[(1+k) + 2(1-k)\cos 2\theta]$$



{when $\theta = 0°$}

$$\sigma_\theta = \sigma_v(3-k) \quad (A)$$

{when $\theta = 90°$}

$$\sigma_\theta = \sigma_v(3k-1) \quad (B)$$

* $k=1$ (Hydrostatic field

$$\sigma_H = \sigma_v$$

$$\sigma_r = \sigma_v\left(1-\frac{a^2}{r^2}\right)$$

$$\sigma_\theta = \sigma_v\left(1+\frac{a^2}{r^2}\right)$$

$$\tau_{r\theta} = 0$$

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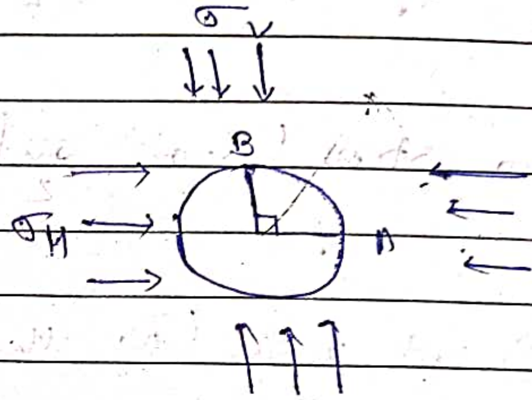
Q. A tunnel is constructed in a biaxial far field stress as shown in figure. If the ratio of tangential stress measured at the boundary points A and B is 3:1, find the value of K.

Sol^n :-

$$\frac{(\sigma_\theta)_A}{(\sigma_\theta)_B} = \frac{3}{1}$$

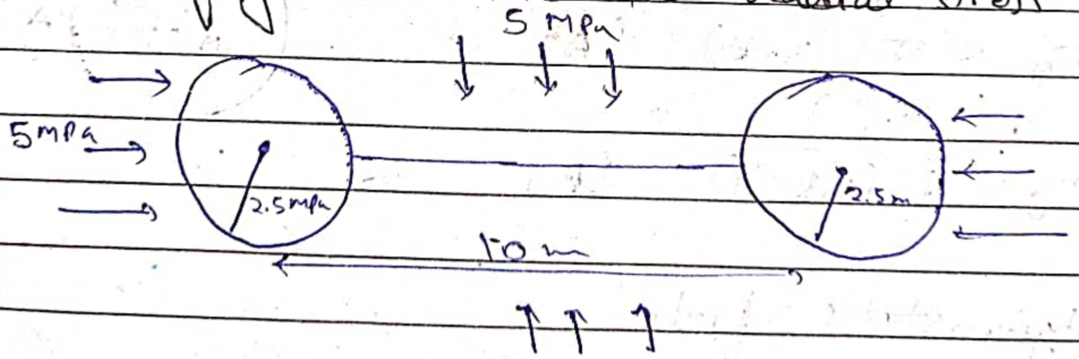


$$\frac{\sigma_V(3-k)}{\sigma_V(3k-1)} = \frac{3}{1}$$

Solving we get = $k = 0.6$

Q. In a hydrostatic stress field, point A is the middle of two circular openings as shown in the figure. Find the radial stress at point A.

Ans



It is a case of hydrostatic stress field as $\sigma_V = \sigma_H$, so $k = 1$. Radial stress ⇒

$$\sigma_r = \sigma_V\left(1 - \frac{a^2}{r^2}\right)$$

Total radial stress at point A

$= (\sigma_r)_t + (\sigma_r)_b = 2\times 5\left(1 - \left(\frac{2.5}{5}\right)^2\right)$

$= 2 \times 5 \times \frac{3}{4}$

$= 7.5$ MPa

Q. Calculate the radial (σ_r), tangential (σ_θ) and shear stress ($\tau_{r\theta}$) on the circumference of a circular opening of 3 m diameter at $\theta = 0°$ and $\theta = 90°$. The unit ~~length~~ weight of rock is 0.028 MN/m^3, Poisson's ratio is 0.3 and the depth of excavation is 300m

Soln:- $\sigma_v = \gamma H = 300 \times 0.028$ MPa
$= 8.4$ MPa

$K = \frac{\mu}{1-\mu}$

$\sigma_r = \tau_{r\theta} = 0$

$\sigma_\theta = \sigma_v(3-K)$ at $0° = 8.4 \times (3 - 0.43)$ MPa

$= \sigma_v(3K-1)$ at $90° = 8.4\,(1.29 - 1)$ MPa

Q7) A circular opening of 3m diameter has been driven at a depth of 450 m from surface. The unit weight of overlying strata is 6 kN/m^3. Compressive strength is 25 MPa and tensile strength is 3 MPa. What is the state of stress condition (safe or unsafe) at the boundary of the o- for $K = 0.3$

$\overline{3.14 \times 350}$

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Soln:- At boundary radial stress is present.

At 0° the opening is under compression whereas
At 90° it is under tension.

$$\sigma_v = 6 \times 450 \text{ kPa} = 2.7 \text{ MPa}$$
$$(\sigma_\theta)_0 = \sigma_v(3-k) = 2.7 \times 2.7 \text{ MPa} = 7.29 \text{ MPa}$$
$$(\sigma_\theta)_{90} = \sigma_v(3k-1) = 2.7(0.9-1) = -0.27 \text{ MPa}$$

Factor of Safety at $\theta = 0° = \frac{25}{7.29} = 3.43$

Factor of safety at $\theta = 90° = \frac{3}{0.27} = 11.11$

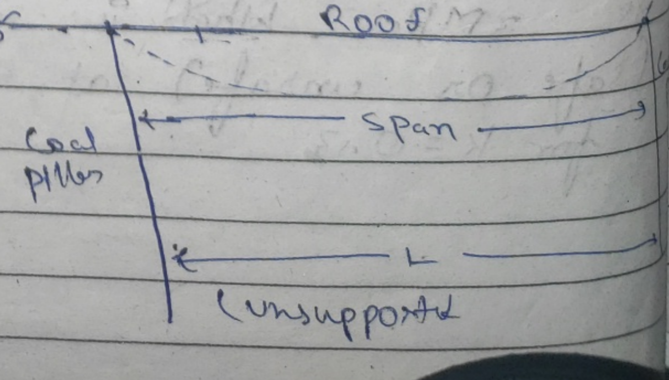
Opening safe

DETERMINATION OF SAFE SPAN OF ROOF:-
i.e length of unsupported part of roof

① Single layer roof:- The overlying strata consists of a single rock.

$$\boxed{\eta_{max} = \frac{\gamma L^4}{32 E t^2} \text{ (m)}} \rightarrow \text{Reduction in height}$$

γ = wt density of rock (kN/m³)
L = span of roof layer



$$71.50 = \pi d V$$
$$\frac{71.50}{3.14 \times 350} = d$$

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Soln:- At boundary radial stress is present

At 0° the opening is under Compression where as
At 90° it is under tension.

$$\sigma_v = 6 \times 450 \text{ kPa} = 2.7 \text{ MPa}$$
$$(\sigma_\theta)_0 = \sigma_v(3-k) = 2.7 \times 2.7 \text{ MPa} = 7.29 \text{ MPa}$$
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Opening safe

1) DETERMINATION OF SAFE SPAN OF ROOF:-
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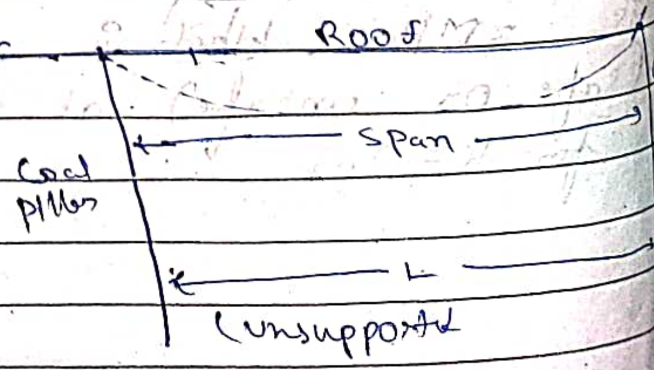

γ = wt density of rock (kN/m^3)
l = span of roof layer

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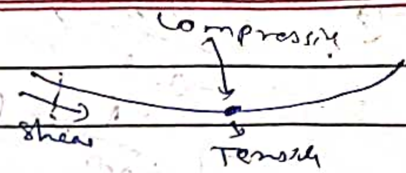

E = young's modulus (MPa)
T = Thickness of roof layer (m).

→ The maximum deflection occurs at the centre of span (or layer).

→ The maximum compressive, tensile and shear stress occur at the ends of the layer

→ At the centre of the span, shear stress is zero and tensile stress is half of its maximum value.

→ The maximum shear stress varies directly as the span.

→ The maximum tensile stress varies directly as as the square of the span and inversely as thickness of layer

$$\boxed{\frac{\sigma_{max}}{\tau_{max}} = \frac{2L}{3T}}$$

→ The Length of span $L = \sqrt{\frac{2R_o t}{\gamma F_s}}$

R_o = modulus of rupture (tensile str

F_s = Factor of saf

→ In case of inclined roof, $L = \sqrt{\frac{2R_o t}{\gamma F_s \cos\theta}}$

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