GATE – Mining Engineering

(Topic Wise Questions 2007-2017)

Topic: ENGINEERING MATHEMATICS

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GATE SYLLABUS: ENGINEERING MATHEMATICS

Linear Algebra: Matrices and Determinants, Systems of linear equations, Eigen values and Eigen vectors.

Calculus: Limit, continuity and differentiability; Partial Derivatives; Maxima and minima; Sequences and series; Test for convergence; Fourier series.

Vector Calculus: Gradient; Divergence and Curl; Line; surface and volume integrals; Stokes, Gauss and Green's theorems.

Diferential Equations: Linear and non-linear first order ODEs; Higher order linear ODEs with constant coefficients; Cauchy's and Euler's equations; Laplace transforms; PDEs –Laplace, heat and wave equations.

Probability and Statistics: Mean, median, mode and standard deviation; Random variables; Poisson, normal and binomial distributions; Correlation and regression analysis.

Numerical Methods: Solutions of linear and non-linear algebraic equations; integration of trapezoidal and Simpson's rule; single and multi-step methods for differential equations.

2007

Q.1 If the slope of a diagonal of a rectangle is m the slope of the other diagonal is

(A) $\frac{1}{2m}$ (B) $-\frac{1}{2m}$ (C) $\frac{1}{m}$ (D) $-\frac{1}{m}$

Q.2 If the rank of a matrix $\mathbf{A}$ is r, the rank of the matrix $\mathbf{A}^T$ is

(A) r, if and only if $\mathbf{A}^T = \mathbf{A}$
(B) r, for all A
(C) p, where $p \neq r$
(D) $r-1$, where $r \geq 1$

Q.17 The measures of dispersion are

(A) range, variance, and standard deviation
(B) mean, median, and variance
(C) mean, mode, and skewness
(D) mean, range, and variance

Q.18 In a single server queueing model with constant arrival rate, which one of the following probability distributions is followed by the inter-arrival times of the customers at the service facility?

(A) binomial (B) Poisson (C) Weibull (D) exponential

Q.58 The random variable X has the following probability mass function

$$P(4)=\frac{1}{4},\quad P(8)=\frac{1}{4},\quad P(12)=\frac{1}{4},\quad P(16)=\frac{1}{4}.$$

The expected value of X is

(A) 1 (B) 3 (C) 10 (D) 12

Q.59 The time between successive failures (in hours) of a side discharge loader operating in a mechanised underground coal mine are as follows:

62, 58, 54, 50, 52, 60, 58, 57, 50, 53

If the failure data follow an exponential distribution, then reliability of the equipment for a period of 50 hours is

(A) 0.25 (B) 0.40 (C) 0.60 (D) 1.00

Q.21 Two sides of a triangle are represented by vectors $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$. The area (magnitude) of the triangle is

(A) $1/\sqrt{2}$ (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Q.22 The cost of diesel is Rs. $\left(25 + \frac{x}{90}\right)$ per km to drive a dump truck at a speed of x km/hour. The maintenance cost of the truck is Rs. 10 per hour. To minimize the cost per km, the truck speed in km/hour is

(A) 5 (B) 20 (C) 25 (D) 30

Q.23 The functions $f(x)$ and $g(x)$ satisfy $f(x=0) = 3$, $f'(x=0) = -5$, $g(x=0) = 2$ and $g'(x=0) = -10$. The value of $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)_{x=0}$ is

(A) –35.0 (B) –5.0 (C) 0.5 (D) 5.0

Q.58 The random variable X has the following probability mass function

$$P(4) = \frac{1}{4}, \quad P(8) = \frac{1}{4}, \quad P(12) = \frac{1}{4}, \quad P(16) = \frac{1}{4}.$$

The expected value of X is

(A) 1 (B) 3 (C) 10 (D) 12

Q.59 The time between successive failures (in hours) of a side discharge loader operating in a mechanised underground coal mine are as follows:

62, 58, 54, 50, 52, 60, 58, 57, 50, 53

If the failure data follow an exponential distribution, then reliability of the equipment for a period of 50 hours is

(A) 0.25 (B) 0.40 (C) 0.60 (D) 1.00

Q.67 The value of $\mathbf{A}\cdot\mathbf{B}$, if $\mathbf{A}+\mathbf{B}=\begin{bmatrix}1 & -1\\ 3 & 0\end{bmatrix}$ and $\mathbf{A}-\mathbf{B}=\begin{bmatrix}3 & 1\\ 1 & 4\end{bmatrix}$, is

(A) $-4\begin{bmatrix}1 & 1\\ 0 & 3\end{bmatrix}$

(B) $-2\begin{bmatrix}1 & 1\\ 0 & 3\end{bmatrix}$

(C) $\begin{bmatrix}1 & 1\\ 0 & 3\end{bmatrix}$

(D) $-\frac{1}{2}\begin{bmatrix}1 & 1\\ 0 & 3\end{bmatrix}$

Q.68 The values of $f(x)$ at x_0, x_1 and x_2 are 9.0, 12.0 and 15.0 respectively. Using the Simpson's $\frac{1}{3}$ rule, the value of $\int_{x_0}^{x_2} f(x)$, considering an interval of 0.1 is

(A) 1.2 (B) 2.4 (C) 1.6 (D) 1.8

Q.25 The solution of $ye^x dx+(4y+e^x)dy=0$ for $y(0)=-1$ is

(A) $ye^x+2y^2-1=0$

(B) $e^x+y^2x-2=0$

(C) $ye^x-y^2=0$

(D) $xe^x+y^2-1=0$

2008

Q.1 The trace of the following matrix is

$$\begin{pmatrix}2 & 2 & 3\\ 3 & 2 & 3\\ 4 & 1 & 2\end{pmatrix}$$

(A) 6 (B) 7 (C) 8 (D) 9

Q.2 If X is a continuous random variable and $f(x)$ defines its probability density function, then the expected value of X is

(A) $\int_{-\infty}^{+\infty} f(x)dx$ (B) $\sum_{i=-\infty}^{+\infty} x_i$ (C) $\sum_{i=-\infty}^{+\infty} x_i f(x_i)$ (D) $\int_{-\infty}^{+\infty} xf(x)dx$

Q.50 Daily production measured for a period of 50 days in a coal mine exhibits normal distribution with mean 1200 tpd and standard deviation 100 tpd. The 95% confidence interval of daily production (standard normal variable Z at 0.025 level of significance is 1.96) in tpd is

(A) 1200 ± 120.5 (B) 1200 ± 96.0 (C) 1200 ± 39.6 (D) 1200 ± 27.7

Q.51 In an iron ore deposit alumina is distributed with $\mu = 3$ % and $\sigma = 0.5\%$; whereas silica is distributed with $\mu = 2.5$ % and $\sigma = 0.8\%$. The combined alumina and silica (as impurities) has μ and σ, in percentage respectively as

(A) (5.5, 0.94) (B) (5.5, 1.3) (C) (0.5, 0.3) (D) (5.5, 0.62)

Q.52 The inverse of the following matrix is:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(A) $\begin{pmatrix} 16 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0.25 & 0 & 0 \\ 0 & 0.50 & 0 \\ 0 & 0 & 1.00 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 16 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

Q.53 The solution of the following system of linear equations is

$$x+4y+3z=0$$
$$3x+5y+2z=0$$
$$8x+10y+12z=0$$

(A) (0,0,0) (B) (1,-1,1) (C) (2, -1, -2) (D) (-3,0,1)

Q.54 The volume of a cone is given by

$$V = \frac{\pi}{3}\ell^3 \sin^2\theta\cos\theta$$

where, ℓ is the slant height and θ is the semi-vertical angle. The angle (θ), for which the volume of cone becomes maximum is

(A) $\sin^{-1}(\frac{1}{\sqrt{3}})$ (B) $\cos^{-1}(\frac{1}{\sqrt{3}})$

(C) $\cos^{-1}(\sqrt{2})$ (D) $\sin^{-1}(\sqrt{2})$

Q.55 The direction of gradient vector at a point (1, 1, 2) on a surface $S(x, y, z) = x^2+y^2 - z$ is

(A) $\frac{1}{3}(2\mathbf{i} +2\mathbf{j} +\mathbf{k})$

(B) $\frac{1}{3}(-2\mathbf{i} +2\mathbf{j} +\mathbf{k})$

(C) $\frac{1}{3}(2\mathbf{i} -2\mathbf{j} +\mathbf{k})$

(D) $\frac{1}{3}(2\mathbf{i} +2\mathbf{j} -\mathbf{k})$

Q.56 The solution of the differential equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}-4y=0, \text{ is}$$

(A) $y = c_1e^{4x}$

(B) $y = c_1e^{2x}$

(C) $y = c_1e^{x} + c_2e^{-4x}$

(D) $y = c_1e^{-x} + c_2e^{4x}$

Q.57 A force vector $\mathbf{F} = (2\mathbf{i}+3\mathbf{j}-\mathbf{k})$ in N is acting on a point, whose position vector $\mathbf{r} = (\mathbf{i}-\mathbf{j}+6\mathbf{k})$ in m. The magnitude of the torque about the origin in Nm is

(A) 20.85 (B) 21.42 (C) 21.97 (D) 22.27

2009

Q.1 If **A** is an orthogonal matrix, then

(A) $\mathbf{A}^T = \mathbf{A}^{-1}$ (B) $\mathbf{A}^T = -\mathbf{A}^{-1}$ (C) $\mathbf{A} = \mathbf{A}^{-1}$ (D) $\mathbf{A} = -\mathbf{A}^{-1}$

Q.2 In a normal (Gaussian) distribution curve, the area between one standard deviation from mean on either side in percent is

(A) 50 (B) 68 (C) 86 (D) 95

Q.3 A measure of dispersion of a sample data set is

(A) mean (B) median (C) mode (D) standard deviation

Q.4 The value of $\lim_{x\to 2}\left(\frac{2\sqrt{4-x^2}}{5}\right)$ is

(A) $-\frac{2\sqrt{8}}{5}$ (B) 0 (C) $\frac{2\sqrt{8}}{5}$ (D) non-existent

Q.5 $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ represent the unit vectors in the positive x, y and z directions of a Cartesian coordinate system. Using the right-hand rule, $\hat{\mathbf{k}} \times \hat{\mathbf{j}}$ represents

(A) 0 (B) 1 (C) $-\hat{\mathbf{i}}$ (D) $\hat{\mathbf{i}}$

Q.21 The mean of the cubes of the first n natural numbers is

(A) $\frac{n(n+1)^2}{4}$ (B) $\frac{n(n+1)(n+2)}{8}$ (C) $\frac{n^4+1}{n}$ (D) $\frac{n^3}{4}$

Q.22 The sum of the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ is

(A) -3 (B) -1 (C) 1 (D) 3

Q.23 The value of $\nabla \cdot \mathbf{F}$ of a vector $\mathbf{F} = 4x^2\hat{\mathbf{i}} + 3xy^2\hat{\mathbf{j}} + xyz^3\hat{\mathbf{k}}$ at the point (1, 1, 2) is

(A) 24 (B) 26 (C) 30 (D) 32

Q.24 The function $f(x) = x^3(1-x)$ is integrated between 0 and 1 (both inclusive) using closed form method and also by Simpson's $\frac{1}{3}$ rule. The difference in the values obtained from these methods is

(A) 0 (B) $\frac{1}{480}$ (C) $\frac{1}{120}$ (D) $\frac{1}{20}$

Q.41 The following data represent the number of workers suffering from pneumokoniosis in 10 coal mines.

Mine	I	II	III	IV	V	VI	VII	VIII	IX	X
Number	10	16	14	15	14	12	17	13	15	12

The number of mines falling above the 50th percentile in terms of the number of workers suffering from pneumokoniosis is

(A) 2 (B) 3 (C) 4 (D) 5

Q.42 Cause-wise data of injuries in an underground coal mine for a five-year period is given below:

Cause of injury	Number of injuries
Fall of roof	27
Fall of person	22
Rope haulage	17
Explosives	5
Other causes	4

The cumulative probability of injury due to fall of roof and fall of person is

(A) 0.65 (B) 0.50 (C) 0.36 (D) 0.29

Common Data for Questions 51 and 52:

Workmen arrive at a mine workshop to receive tools for maintenance. The inter-arrival time of workmen at the service counter is exponentially distributed with an average time of 10 min. The service time at the counter is also distributed exponentially with a mean time of 6 min.

Q.51 Probability that there is a queue (more than one workman) at the service counter is

(A) 0.24 (B) 0.36 (C) 0.40 (D) 0.60

Q.52 Average time spent by a workman waiting for his turn to be served in min is

(A) 9 (B) 12 (C) 15 (D) 18

2010

Q.4 In a Cartesian coordinate system the vertices of a triangular plate are given by (–2, 1), (3, 4), and (–4, –8). The coordinates of the centre of gravity of the plate are

(A) 3, 4 (B) 7, 12 (C) –1, –1 (D) –3, –4

Q.9 For a mine of production t per year, the total cost of production is given by $at^2 + b$. The revenue from sale is given by ct. If a, b and c, are constants, the breakeven value of t is

(A) $[c \pm \sqrt{\{c^2 - 4ab\}}]/(2a)$ (B) $[\sqrt{\{c^2 - 4ab\}}]/(2a)$

(C) $[-c \pm \sqrt{\{c^2 - 4ab\}}]/(2a)$ (D) $[c \pm \sqrt{\{c^2 + 4ab\}}]/(2a)$

Q.10 The value of the $\lim_{x \to 1}\left[\frac{1 - x^{-1/3}}{1 - x^{-2/3}}\right]$ is

(A) ∞ (B) 1

(C) 0 (D) 1/2

Q.11 Two determinants of order n are multiplied. The order of the resultant determinant is

(A) n (B) $2n$ (C) n^2 (D) $n/2$

Q.12 The partial differential equation, $r\frac{\partial \theta}{\partial r} = \text{constant}$, is a solution for

(A) $\frac{\partial^2 \theta}{\partial r^2} - \frac{1}{r}\frac{\partial \theta}{\partial r} = 0$ (B) $\frac{\partial^2 \theta}{\partial r^2} + \frac{\partial \theta}{\partial r} = 0$

(C) $r^2\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} = 0$ (D) $\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} = 0$

Q.21 The mean and the standard deviation of the grade of iron ore in a deposit are 62% and 5% respectively. The coefficient of variation of the grade in % is

(A) 24.8 (B) 12.4 (C) 8.0 (D) 4.0

Q.22 The variance of failure time (time to failures) of an electric motor in shovel is 1600 hr^2. If the failure time follows an exponential distribution, the expected failure time in hr is

(A) 40 (B) 80 (C) 800 (D) 1600

Q.27 The volume of tetrahedron with vertices at (0,0,0), (1,0,0), (0,1,0) and (0,0,1) is

(A) 1/2 (B) 1/4 (C) 1/6 (D) 1/8

Q.32 The queue of trucks at a crusher plant hopper is known to be M/M/1 queue. The probability that there is no truck to unload is 0.3.
Due to rains the mean service time at the hopper is increased by 30%. As a consequence, the expected number of trucks in the queuing system (including the one possibly unloading) becomes

(A) 10 (B) 12 (C) 14 (D) 16

Q.33 Pull from an underground tunnel blasting is normally distributed with a mean of 100 tonnes and variance 100 $(\text{tonnes})^2$. The probability that the tonnage value from a blast exceeds 110 is

(A) 0.60 (B) 0.80 (C) 0.16 (D) 0.32

Q.40 The value of the given integral is

$$\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{\sin x}{(\sin x + \cos x)}\, dx$$

(A) $\frac{\sin \pi/8}{10}$ (B) $\frac{\pi}{10}$ (C) $\frac{\sin \pi/5}{10}$ (D) $\frac{3\pi}{10}$

Q.41 The probabilities of hitting a target by A and B are 1/3 and 2/5 respectively. A shoots at the target once, followed by B shooting at the target once. The probability of hitting the target is

(A) 2/15 (B) 5/15 (C) 8/15 (D) 9/15

Q.42 The value of k for which the points $(5,5),(k,1),(10,7)$ lie on a straight line is

(A) –5 (B) +5 (C) –2 (D) +2

2011

Q.2 The two vectors are orthonormal, if

(A) vector product is zero and norm of each vector is also zero
(B) vector product is one and norm of each vector is also one
(C) cross product is zero and norm of each vector is one
(D) cross product is one and norm of each vector is zero

Q.3 The value of $\lim_{x\to 0}\frac{1}{x}\left\{\sqrt{1+x}-\sqrt{1-x}\right\}$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.4 The infinite series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$, is

(A) convergent (B) divergent (C) oscillatory (D) semi-convergent

Q.5 The largest area of a rectangular shaft for a given constant perimeter is obtained when length is

(A) 2.5 times of breadth
(B) 1.5 times of breadth
(C) 2 times of breadth
(D) equal to breadth

Q.26 For an oil exploration drilling, chance of striking an oil reservoir is 1 out of 15. If an oil exploration company decides to explore 5 sites, the probability of striking at least one successful oil reservoir is

(A) 0.292 (B) 0.250 (C) 0.034 (D) 0.0024

Q.27 Product of the eigen values of the matrix A is

$$A=\begin{pmatrix}3 & 2 & 5\\ 2 & 2 & 1\\ 1 & 5 & 4\end{pmatrix}$$

(A) 6 (B) 8 (C) 10 (D) 35

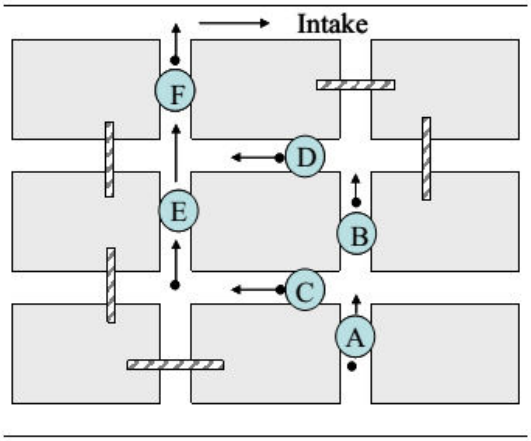
Q.28 For the equation $\frac{dy}{dx}=2x+3y$, the value of y at $x=0.1$ in one step using Runge-Kutta fourth order method for the condition $y=1$ when $x=0$, is

(A) 0.3608 (B) 1.2508 (C) 1.3608 (D) 1.4625

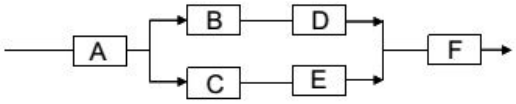
Q.29 Value of the integral $\int_0^1\sqrt{\frac{1+x}{1-x}}dx$ is

(A) $\frac{\pi}{2}-1$ (B) $\frac{\pi}{2}+1$ (C) $\pi-1$ (D) $\pi+1$

Q.46 A sudden increase of CO incidence has occurred in an underground mine section. A man at point A starts to run out to the main intake of the mine where he will be safe. Refer figure below for the mine section and the logic diagram. The probabilities that he will successfully cross the gallery sections A, B, C, D, E, and F are 0.9, 0.8, 0.7, 0.8, 0.7 and 0.9 respectively. The probability that he will successfully reach the main intake is



Section of the mine



Series-parallel logic diagram

(A) 0.40 (B) 0.51 (C) 0.66 (D) 0.77

2012

Q.2 The coefficient of variation of a dataset is measured by

(A) $\frac{\text{mean}}{\text{standard deviation}}$ (B) $\frac{\text{mean}}{\text{variance}}$

(C) $\frac{\text{standard deviation}}{\text{mean}}$ (D) $\frac{\text{variance}}{\text{mean}}$

Q.3 The value of $\int_0^1 \sin^{-1}(\cos x)\,dx$ is

(A) $\frac{(\pi-1)}{2}$ (B) $\frac{(\pi+1)}{2}$ (C) $\frac{(2\pi+1)}{2}$ (D) $\frac{(2\pi-1)}{2}$

Q.4 Assuming $\sin(1)=0.841$ and $\sin(3)=0.141$, the Lagrangian linear interpolating polynomial, for the function $f(x)=\sin(x)$ defined on the interval [1, 3] and passing through the end points of the interval, is

(A) $-0.35x+1.19$
(B) $-3.05x+11.92$
(C) $-35.00x+119.10$
(D) $-40.50x+219.19$

Q.6 The 2nd order differential equation having a solution $y = (A/x) + B$, where A and B are constants, is

(A) $\frac{d^2y}{dydx} + \frac{2}{x}\frac{dy}{dx} = 0$

(B) $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} = 0$

(C) $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{2}{x}\frac{dy}{dx} = 0$

(D) $\frac{d^2y}{dydx} + \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Q.31 An iron ore mine recorded an average of 3 accidents per month. The number of accidents is distributed according to Poisson distribution. The probability that there will be exactly 2 accidents per month is

(A) 0.22 (B) 0.30 (C) 0.43 (D) 0.67

Q.37 The angle between the tangents to the curve $\vec{R} = t^2\hat{i} + 2t\hat{j}$ at the point $t = \pm 1$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Q.43 The cofactor matrix of $P = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ is

(A) $\begin{bmatrix} 21 & -5 & 2 \\ 2 & 21 & -5 \\ -5 & 2 & 21 \end{bmatrix}$ (B) $\begin{bmatrix} 21 & 2 & -5 \\ 2 & 7 & 15 \\ -5 & 21 & 2 \end{bmatrix}$ (C) $\begin{bmatrix} -5 & 2 & 21 \\ 15 & 7 & 2 \\ -2 & 21 & -5 \end{bmatrix}$ (D) $\begin{bmatrix} 15 & 7 & 2 \\ -5 & 2 & 21 \\ -2 & 21 & -5 \end{bmatrix}$

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:
A mining project is composed of five activities whose three time estimates in months are given below:

Activity	Estimated duration (months)		
	Optimistic time	Most likely time	Permissible time
1-2	1	1	7
1-3	2	5	8
2-4	1	1	7
3-4	2	5	14
4-5	3	6	15

Q.52 The expected duration of the mining project in months is
(A) 5 (B) 16 (C) 18 (D) 29

Q.53 The standard deviation of the project length in months is
(A) 2 (B) 3 (C) 6 (D) 9

2013

Q.2 If the transpose of a matrix is equal to its inverse, then the matrix is
(A) symmetric (B) orthogonal (C) skew symmetric (D) singular

Q.12 Events A and B are independent but NOT mutually exclusive. If the probabilities P(A) and P(B) are 0.5 and 0.4 respectively, then P($A \cup B$) is
(A) 0.6 (B) 0.7 (C) 0.8 (D) 0.9

Q.14 Identify the correct statement for a 'normal distribution'.
(A) Mean is greater than mode but less than median
(B) Mean is less than mode but greater than median
(C) Mean is greater than mode and median
(D) Mean, median and mode are equal

Q.16 The number of ways in which the letters in the word MINING can be arranged is
(A) 90 (B) 180 (C) 360 (D) 720

Q.34 If the following linear system of equations has non-trivial solutions

$$\begin{aligned} px + y + z &= 0 \\ 2x + y - 2z &= 0 \\ x + 2y - 3z &= 0 \end{aligned}$$

the value of p is

(A) 1 (B) 0 (C) -1 (D) -7

Q.37 The value of $\int_0^{\pi/2} \log(cosx)\, dx$ is

(A) $-\frac{\pi}{2} log2$ (B) $-\frac{\pi}{4} log2$ (C) $\frac{\pi}{2} log2$ (D) $\frac{\pi}{4} log2$

Q.42 Given the following differential equation

$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = 0$$

the general solution is

(A) $y = Ae^{4x} + Be^{-3x}$
(B) $y = Ae^{-4x} + Be^{-3x}$
(C) $y = Ae^{3x} + Be^{-4x}$
(D) $y = Ae^{4x} + Be^{3x}$

Q.47 A wire of length L is cut into two pieces to construct a circle and an equilateral triangle such that the combined area is minimum. The length of the wire used to construct the circle is

(A) $\frac{\sqrt{3}\pi L}{9+\sqrt{3}\pi}$ (B) $\frac{9L}{9+\sqrt{3}\pi}$ (C) $\frac{L}{2}$ (D) $\frac{18L}{9-\sqrt{3}\pi}$

Statement for Linked Answer Questions 52 and 53:

Economic analysis of an iron ore deposit reveals that the net value of the ore is related to the grade mined as shown in the table.

Grade (%Fe)	Net value of ore (Rs/tonne)
64.5	3200
60.2	1800

Q.52 Assuming linear relationship between the net value and grade, the break-even cut-off grade in % Fe is

(A) 52.2 (B) 54.7 (C) 58.0 (D) 62.2

Q.53 Assuming that the grade follows normal distribution with mean 62.7%, and standard deviation 10.0% (A portion of the standard normal distribution table is given below),

z	0.00	0.01	0.02	0.03	0.04
0.6	0.72575	0.72907	0.73237	0.73565	0.73891
0.7	0.75803	0.76115	0.76424	0.76730	0.77035
0.8	0.78814	0.7 9103	0.79389	0.79673	0.79954
0.9	0.81594	0.81859	0.82121	0.82381	0.82639
1.0	0.84134	0.84375	0.84613	0.84849	0.85083

the percentage of waste in the deposit based on the break-even cut-off grade is

(A) 78.8 (B) 71.2 (C) 28.8 (D) 21.2

2014

Q.11 Solution of the differential equation $\frac{dy}{dx} = ky$ follows exponential decay (where k is a constant) for $x \in [0,\infty]$ if

(A) $k > 0$ (B) $k < 0$ (C) $k = 0$ (D) $k = e$

Q.12 The value of k for which the vectors $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$ and $\mathbf{b} = k\mathbf{i} + 4\mathbf{j}$ are orthogonal to each other is ___

Q.14 The occurrence of head in a single toss of an unbiased coin is given by a random variable X. The variance of X is _____

Q.15 The divergence of the vector $\mathbf{v} = (x+y)(-y\mathbf{i} + x\mathbf{j})$ is

(A) $y - x$ (B) $x - y$ (C) $x^2 - y^2$ (D) $y^2 - x^2$

Q.16 The $\lim_{x \to 0} \frac{|x|}{x}$ is

(A) -1 (B) 0 (C) 1 (D) non-existent

Q.18 Given k is the thermal conductivity, ρ is density and c is specific heat of a rock sample, the thermal diffusivity of the rock sample is

(A) $\frac{k\rho}{c}$ (B) $\frac{\rho c}{k}$ (C) $\frac{kc}{\rho}$ (D) $\frac{k}{\rho c}$

Q.36 For an explosives company, the probability of producing a defective detonator is 0.02. The probability that a lot of 50 detonators produced by the company contains at most 2 defective detonators is _____

Q.37 The area enclosed by the curves $y = x^2$ and $y = x^3$ for $x \in [0, \infty]$ is

(A) 1/12 (B) 1/6 (C) 1/2 (D) 1

Q.38 The value of a, for which the function below is continuous at $x = 1$ is

$$f(x) = \begin{cases} 2x + ax^2, & x \le 1 \\ 4x + 3, & x > 1 \end{cases}$$

(A) -5 (B) 0 (C) 5 (D) 10

Q.39 The sum of the infinite series $a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} + \cdots$ for $|r| < 1$ is

(A) $a(1+r)$ (B) $a(1-r)$ (C) $\frac{a}{1+r}$ (D) $\frac{a}{1-r}$

Q.54 The failure and the repair rates of a shovel are 0.06 hr^{-1} and 0.04 hr^{-1} respectively. The availability of the shovel in percentage is _____

2015

Question Number : 20 Question Type : MCQ

In a binomial distribution, the probability of success $p \to 0$ and number of trials $n \to \infty$ such that $\lambda = np$ approaches to a finite value. The variance of the distribution is

(A) $np\lambda$ (B) $n\lambda$ (C) $p\lambda$ (D) λ

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 21 Question Type : NAT

For a function $f(x)$, it is given that $f(0) = 2$ and $f'(0) = 4$. Ignoring all other higher order derivative terms, the value of $f(0.5)$ is _____

Correct Answer :

4

Question Number : 22 Question Type : MCQ

The two sides of a parallelogram are given by the vectors $\mathbf{A} = 2\hat{i} - 3\hat{j}$ and $\mathbf{B} = 3\hat{i} + 2\hat{j}$. The area of the parallelogram is

(A) 13 (B) 12 (C) 10 (D) 5

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 29 Question Type : MCQ

The matrix $A = \begin{bmatrix} -4/6 & 2/6 & 4/6 \\ 4/6 & 4/6 & 2/6 \\ 2/6 & -4/6 & 4/6 \end{bmatrix}$ is

(A) orthogonal (B) diagonal (C) skew-symmetric (D) symmetric

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 41 Question Type : MCQ

The value of $\int_0^4 \sqrt{16 - x^2}\, dx$ *is*

(A) 12.57 (B) 50.24 (C) 25.12 (D) 3.14

Options :

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

Question Number : 57 Question Type : NAT

Rock bolts have length $L = (150 + X)$ cm, where X is a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{4}(1 - 3x), & \text{if } -2 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

If 95% of the bolt lengths (L) lie in the interval $150 - c$ cm to $150 + c$ cm, the value of c is _____

Correct Answer :

1.88 to 1.92

Question Number : 56 Question Type : MCQ

Acceleration of a particle moving in a straight line is expressed by

$$\frac{d^2s}{dt^2} = 2t$$

where, s denotes distance (m) and t, time (s). At time $t = 0$, the distance and velocity of the particle are 0 m and 3 m/s respectively. The distance travelled by the particle in m after 3 s is

(A) 3 (B) 6 (C) 9 (D) 18

Options :

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

Question Number : 58 Question Type : NAT

The properties for a bivariate distribution of two random variables X and Y are given below.

$$E(X) = 24,\ E(Y) = 36,\ E(X^2) = 702,\ E(Y^2) = 1524,\ E(XY) = 1004$$

The correlation coefficient between X and Y is _____

Correct Answer :

0.8 to 0.85

2016

Q. 1 – Q. 25 carry one mark each.

Q.1 The differential of the equation, $x^2 + y^2 = 1$, with respect to x is

(A) $-x/y$ (B) x/y (C) $-y/x$ (D) y/x

Q.2 If $[A][B] = [I]$ then

(A) $[B] = [A]^T$ (B) $[A] = [B]^T$ (C) $[B] = [A]^{-1}$ (D) $[B] = [A]$

Q.3 $X^4 + C$ is the general integral of

(A) $3\int x^3 dx$ (B) $\frac{1}{4}\int x^3 dx$ (C) $\int x^3 dx$ (D) $4\int x^3 dx$

Q.4 *Sinh* (x) is

(A) $\frac{e^x - e^{-x}}{4}$ (B) $\frac{e^x - e^{-x}}{2}$ (C) $\frac{e^x + e^{-x}}{2}$ (D) $\frac{e^x + e^{-x}}{4}$

Q. 26 – Q. 55 carry two marks each.

Q.26 Equations of two planes are $z = 4$ and $z = 4 + 3x$. The included angle between the two planes in degrees, is _____

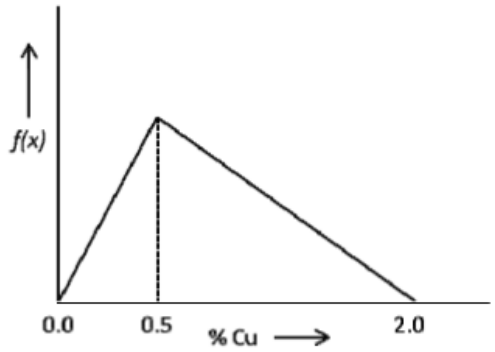
Q.27 A force $\vec{P} = 2\hat{i} - 5\hat{j} + 6\hat{k}$ acts on a particle. The particle is moved from point A to point B, where the position vectors of $\vec{A}$ and $\vec{B}$ are $6\hat{i} + \hat{j} - 3\hat{k}$ and $4\hat{i} - 3\hat{j} - 2\hat{k}$ respectively. The work done is _____

Q.28 The value of x in the simultaneous equations is _____

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

Q.29 Two persons P and Q toss an unbiased coin alternately on an understanding that whoever gets the head first wins. If P starts the game, then the probability of P winning the game is _____

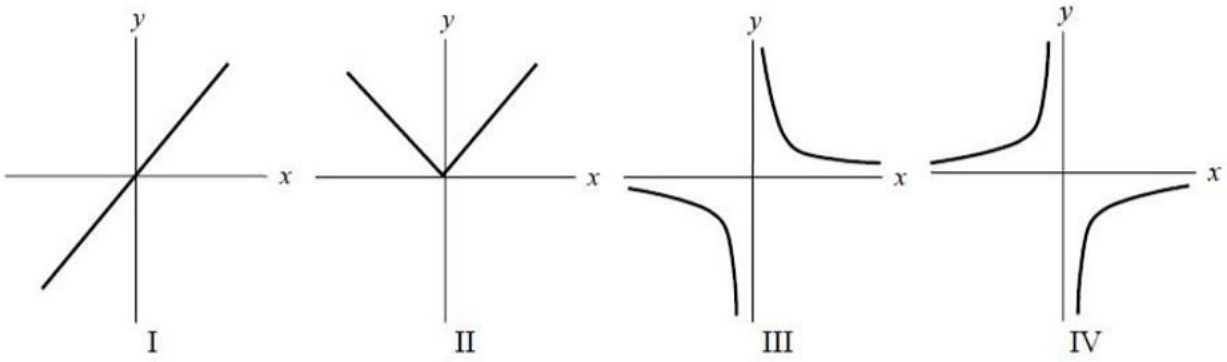
Q.52 Copper grade distribution in an ore body has the probability density function, $f(x)$, as shown in the figure. The average grade of the deposit, in % Cu, is _____.



2017

Question Number : 1 **Correct : 1 Wrong : -0.33**

Which one of the following plots represents the relationship $xy = c$, where c is a positive constant



(A) I (B) II (C) III (D) IV

Question Number : 2 **Correct : 1 Wrong : -0.33**

$F(y)$ and $f(y)$ are the probability distribution function and density function respectively of a continuous variable Y in the interval $(0, \infty)$. Which one of the following is TRUE?

(A) $F(y) = \int_y^{\infty} f(x)dx$

(B) $F(y) = \int_o^{y} f(x)dx$

(C) $F(y) = \frac{df(y)}{dy}$

(D) $F(y) = 1 - f(y)$

Question Number : 4 **Correct : 1 Wrong : -0.33**

The value of $\lim_{x\to\infty} \left(\frac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{2x-1}}$

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 1

(D) 0

Question Number : 26 **Correct : 2 Wrong : 0**

The y intercept of the tangent of curve $y = x^3 - x^2 + x - 1$ at $x = 1$ is _____

Question Number : 27 **Correct : 2 Wrong : 0**

A rectangle has two of its corners on the x axis and the other two on the parabola $y = 12 - x^2$. The largest area of the rectangle is _____

Question Number : 28 **Correct : 2 Wrong : 0**

The area of cross-section (x) of four rock samples and the respective applied loads (y) at failure under uniaxial loading are given below:

x (cm^2)	7	10	13	16
y (kN)	35	45	60	80

If the best fit line $y = 4.88x$ represents the above data, the coefficient of determination (R^2) of the best fit line is _____

Question Number : 34 **Correct : 2 Wrong : 0**

Consider the following linear programming problem:

Maximize $Z = 6X + 10Y$
Subject to $X \leq 4$
$Y \leq 6$
$3X + 2Y \leq 18$
$X \geq 0$, $Y \geq 0$

The maximum value of the objective function is _____

Question Number : 46 **Correct : 2 Wrong : -0.66**

If the rank of the following matrix is less than 3, the values of x are

$$A = \begin{bmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{bmatrix}$$

(A) $1, -1/2$
(B) $1, 1/2$
(C) $2, -1/4$
(D) $2, -3/4$

Question Number : 53 **Correct : 2 Wrong : 0**

In a book of 600 pages, there are 60 typographical errors. Assuming Poisson distribution for the number of errors per page, the probability of no errors in randomly chosen 4 pages is _____